The importance of end effects in the measurement of moduli of highly anisotropic materials

The purpose of this note is to draw attention to a potential source of error in the measurement of moduli of highly anisotropic materials As the issue is quite general irrespective of the material nature of the test sample, as long as it is highly anisotropic and of the testing method used we thought it appropriate to highlight it in a note for all those concerned with the practicability of determining moduli and drawing conclusions therefrom. The underlying solid mechanics will only be outlined; a quantitative treatment of it is contained in more comprehensive publications elsewhere.

The source of error implied relates to end effects which are defined by St Venant's principle. This, in simple terms, means that end effects in the torsion or tension of cylinders, or in the bending of beams, die away within a length approximately equal to the maximum cross-sectional dimension and has allowed investigators largely to ignore end effects when making measurements of elastic constants. However, Horgan [1] has recently shown theoretically that in highly anisotropic materials the decay length" can be considerably greater than the maximum cross-sectional dimension.

The effect of this upon measurements by torsional methods of the shear modulus of an anisotropic polymer has already been encountered recently by Folkes and Arridge [2] in work on a rather special type of co-polymer where it was found that the constraints at the two ends increased the apparent modulus for samples of length/width ratio less than about 80, until for samples where the ratio was less than 30 the apparent modulus was double the long-sample value. In the following we define aspect ratio to mean the ratio length/maximum cross-sectional dimension.

In tensile specimens gripped at the surface at each end the effect should be reversed. The load is preferentially transferred along the surface of the specimen leaving the regions near the axis relatively unstrained. The effective Young modulus therefore estimated by movement of the points of application of load, or by surface strain gauges, will be lower than the true value. As the aspect ratio increases the load becomes more uniformly diffused across the entire cross-section and the effective modulus will approach the true value. An exact treatment of these effects is difficult, although numerical methods [3] enable an approximate estimate of elastic moduli to be made for prescribed loadings. It is, therefore, important experimentally to ensure that specimen aspect ratios are large enough for the extended end effects in anisotropic materials to be neglected.

Out of a number of examples we shall illustrate the effect of the aspect ratio on apparent moduli in the case of the tensile modulus of very highly drawn polyethylene, in view of the fact that this type of material has recently acquired some topical interest [4]. The material was high density polyethylene Rigidex 50 drawn to a draw ratio of 28 by methods similar to those of Capaccio and Ward [4] and detailed as S1 type by Barham and Keller [5]. The drawn samples were strips 0.48 mm wide and 0.24 mm thick. The aspect ratio was varied by cutting out test specimens of different length in the range of 3.9 to 111 mm, from the same strip for a given test series to avoid batch variations. The original sample width was thus left unaltered, since cutting of the specimens into narrower pieces would have introduced damage. According to the predictions of theory the important factor is the ratio of length to maximum cross-sectional dimension, thus doubling the latter should double the persistence length of end effect. The above procedure, therefore, serves our purpose while admittedly testing these predictions by altering both length and width could be a useful future objective.

The samples were secured at each end by Araldite into crimped tubes which were, in turn, gripped conventionally in an Instron machine. Stress was determined from the load–elongation curve of the Instron, the cross-section being measured optically and checked independently by weighing.

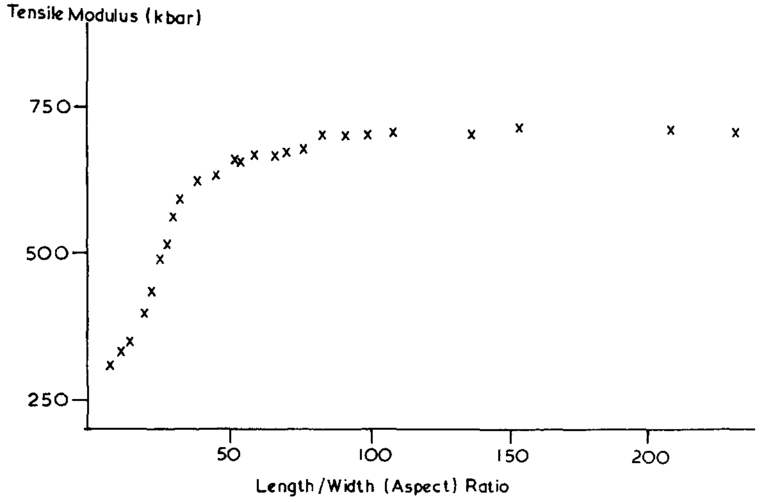

Figure 1 The dependence of apparent tensile modulus on length/width ratio for a 28X drawn linear polyethylene film. Measurements of tensile modulus were made at 0.1% strain and at a strain-rate of 10^{-4} sec^{-1} (c.f. [5]).

The displacement in the sample was estimated from the chart displacement of the Instron*. This displacement includes the following factors: true sample displacement, displacement of the Araldite cement, displacement in the grips, softness of the load cell and machine mechanoelectronic linkage between cross-head and chart. Calibration was made by using a steel wire of length 58 cm, i.e. much longer than the specimens, with fiduciary marks, mounted in an identical manner to that used for the samples. The strain in this wire can be assumed to be homogeneous. Photographs were, therefore, taken of the fiduciary marks at various cross-head displacements (this was possible due to the large displacements that could be used for calibration purposes) and the correction factors so obtained used when estimating sample overall mean strain (that is, the equivalent homogeneous strain in the sample). Three determinations of the secant modulus at 0.1% equivalent strain were made using the same strain-rate, namely 10^{-4}sec^{-1} in each case. The reproducibility of modulus measurements, taking account of all sources of error, is estimated as within 10% of the mean for any one length.

Fig.1 shows the dependence of the apparent tensile modulus obtained by the above methods on the sample aspect ratio. It is seen that the apparent tensile modulus increases with aspect ratio by up to a factor of three reaching a constant value at an aspect ratio of about 80 to 100. This is in accord with the theoretical expectations. It is apparent that care must be taken to ensure that samples have a sufficiently large aspect ratio for end effects to be ignorable and this can mean a ratio of 100 to 200 for highly orientated materials. Where such ratios are not attainable the elastic modulus is not measurable at the present state of knowledge and consequently, without information on the dimensions of samples, it may be difficult to compare published values obtained on such samples.

The requirement of a modification to St Venant's principle when anisotropic materials are involved profoundly affects all methods of measuring elastic moduli besides those referred to above. For example, we have observed effects similar to those in Fig. 1 for the flexure of beams by three-point bending [6] illustrating that the effects of points of load application cannot be neglected. A

*The displacement would of course, be best measured directly at the surface by means of fiducial marks or a surface strain gauge. However, both these methods are impractical for these specimens. The use of fiducial marks, because at the small displacements (~ 0.1% strain) involved and the comparatively short lengths used to show the effect, the displacements of such marks would be at the limits of optical resolution. The use of strain gauges is impractical due both to the hardness of such gauges compared to the specimen and to the small width (~ 0.5 mm) of the specimen. While attempts to devise an absolute method of surface strain determination are in hand, for the present note the indirect method described above was used.

comprehensive study of these problems is currently being made. It should, however, be apparent even on the basis of the present note, that the issue in question is of tangible consequence for the design and standardization of modulus experiments and for the interpretation of data obtained.

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ANNOUNCEMENT

American Society of Biomechanics

To stimulate research and encourage communications and co-operation in the field of biomechanics, the formation of the *American Society of Biomechanics* is proposed. Regular meetings will be structured to allow an exchange of information and discussion among those engaged in applying the principles of mechanics to biological problems. A working committee consisting of Richard A. Brand (Medicine); Don B. Chaffin (Ergonomics); F. Gaynor Evans (Biology); James G. Hay (Physical Education); and Albert B. Schultz (Engineering) is investigating the details of forming the Society. Persons actively engaged in biomechanics research and publication are asked to indicate their interest by contacting Gary L. Soderberg, Physical Therapy, The University of Iowa, Iowa City, Iowa 52242, USA.